

Subject: Mathematics
Syllabus for Paper-II (Coursework of Ph.D.)

Unit-1: Real Analysis

1. Elements of set theory

Countable and uncountable sets, Infinite sets, Cardinal numbers and their arithmetic, Schroeder-Bernstein theorem, Cantor's theorem and the continuum hypothesis, Poset, Axiom of choice, Housdorff maximality principle, Tukey's theorem, Zorn's lemma, well ordering theorem.

2. Sequence and series of real numbers

Sequence, Limit of a sequence, Convergence of sequence, limit superior, limit inferior of a sequence of real numbers, Cauchy sequence, Series, Convergence of Series, tests for Convergence of Series.

3. Riemann Integral

Darboux's upper and lower sums, lower and upper integrals, Riemann integral criterion for Riemann integrability, Properties of integral functions, Fundamental theorem of calculus, the integral as a limit of sum.

4. Improper Integrals.

Improper integrals, types of improper integrals, Convergence of improper integrals, test integrals and some tests for convergence. Gamma and Beta functions.

Unit-2: Complex Analysis

1. Complex functions as mappings, Analytic function, Bilinear transformation, Conformal mappings.

2. Power series representation of Analytic functions, Radius of convergence, Cauchy's estimate, Zero's of analytic functions, Integration in complex plane. Fundamental theorem of algebra, Maximum modulus theorem.

3. Singularities, Classification, Poles and essential singularities, Singular point, Laurent series, Residues and evaluation of integrals.

4. Entire functions, Weierstrass theorem.

Unit-3: Algebra

1. Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems.

2. Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain.

3. Polynomial rings and irreducibility criteria.

4. Fields, finite fields, field extensions, Galois Theory.

Unit-4: Linear Algebra

1. Vector Spaces, Subspaces, Linear dependence and Linear independence of Vectors, System of linear equations, Basis and Dimension of a Vector Space, Quotient Space.
2. Linear transformations, Range and Null Space of a linear transformation, Trace of a matrix, Eigen values and Eigen vectors, The Cayley-Hamilton theorem, Minimal polynomial.
3. Modules, Submodules, Free modules, Faithful modules, Direct sum of modules.
4. Local rings and Modules, Jordan Canonical forms.

Unit-5: Ordinary and Partial Differential Equations

1. **Second order linear equations:** Fundamental solution of the homogeneous equation, linear independence, reduction of order, linear equations with constant coefficients, method of undetermined coefficients, method of variation of parameters.
2. **System of first order equations:** solution of linear system, basic theory of system of first order linear equations, linear homogeneous system, fundamental matrices, non homogeneous linear system.
3. **Elementary Partial Differential Equations:** Definitions, equations solvable by direct integration, nonlinear equations of first order, Charpits method.
4. **Single first order PDEs:** Semilinear and quasilinear equations, characteristics Cauchy problem, general solutions. Classification of second order PDEs.

Unit-6: Mathematical Methods

1. **Laplace Transforms:** Definition, notation, sufficient condition for existence, properties of L.T., Initial value and final value theorems. Methods of finding L.T., Definition of Inverse Laplace transform, uniqueness and properties, Convolution theorem. Methods of finding inverse L.T., Applications of L.T. to LDE and PDE.
2. **Fourier Transforms:** The Fourier Integral, complex form of Fourier Integrals and Fourier Integral theorem; Fourier transforms; Fourier Cosine and Sine Transforms, Finite Fourier transforms, Convolution theorem, Parsval's Identity .
3. **Linear Boundary value problems:** Introduction, wave equation, heat equation and Laplace's equation in Cartesian, Cylindrical and Spherical co-

4. Seymour Lipschutz : *Theory and Problems of linear Algebra*, Schaum's outline series.
5. I. S. Luthar and I. B. S. Passi: *Algebra, Volume 3: Modules*, Narosa Publishing House.

Unit-5:

1. William E. Boyce and Richard C. DiPrima: *Elementary Differential Equations and Boundary Value Problems*, Second Edition, John Wiley and Sons, Inc., New York.
2. Earl A. Coddington: *An Introduction to Ordinary Differential Equations*, Prentice-Hall India, 2001.
3. H. K. Dass: *Advanced Engineering Mathematics*, S. Chand and Company Ltd., New Delhi 1999.
4. Fritz John: *Partial Differential Equations*, Forth edition, Springer International edition, New York 2009.
5. Robert C. McOwen: *Partial Differential Equations Methods and Applications*, Pearson Education Inc., Indian branch Delhi, 2004.
6. Phoolan Prasad and Renuka Ravindran: *Partial Differential Equations*, New age International Publishers, New Delhi 2005.
7. George F. Simmons: *Differential Equations with Applications and Historical Notes*, Tata McGraw-Hill Publications, New Delhi 1985.

Unit-6:

1. J. P. Chauhan: *Differential and Difference equations*, Garg Publishing House.
2. R. V. Churchill and J. W. Brown: *Fourier Series and Boundary value problems*, Mc Graw-Hill international.
3. Goyal and Gupta: *Laplace transforms and Fourier transforms*, Pragati Prakashan.
4. Murray J. Spiegel: *Laplace Transforms*, Schaum's outline Series.

ordinates. Principle of superposition, series solutions, Fourier Series, Solution of Linear boundary value problem by separation of variables, types of initial value problems and general solution of partial differential equations. Sturm-Liouville problems.

4. **Difference Equations:** Definition, order, linear difference equation, formation, existence and uniqueness theorem, solution of the equation, general solution of the homogenous difference equations and non-homogenous linear difference equations.

Reference Books:

Unit-1:

1. Savita Arora and S. C. Malik: *Advanced Analysis*, New Age International Ltd.
2. Robert G. Bartle and Donald R. Sherbert: *Introduction to Real Analysis*, John Wiley and Sons.
3. R. R. Goldberg: *Methods of Real Analysis*, New Age International Ltd.

Unit-2:

1. J. B. Conway: *Functions of one complex variable*, Springer-Verlag.
2. S. Ponnusamy: *Foundation of Complex Analysis*, Narosa Publishing House.
3. Murray R. Spiegel: *Complex Variables*, Schaum's Outline Series.

Unit-3:

1. David S. Dummit and Richard M. Foote: *Abstract Algebra*, John Wiley and Sons.
2. J. B. Fraleigh: *A first course in Abstract Algebra*, Narosa publishing House.
3. I. N. Herstein: *Topics in Algebra*, Wiley – Eastern.
4. V. K. Khanna and S. K. Bhambri: *Abstract Algebra*, Vikas Publication.
5. I. S. Luther and I. B. S. Passi: *Algebra-Groups- Volume 1*, Narosa Publishing House.

Unit-4:

1. N.S.Gopalkrishna : *University Algebra*, Wiley-Eastern 1988.
2. N.S.Gopalkrishna : *Commutative Algebra*, Oxonlan Press 1988.
3. Serge Lang : *Linear Algebra*, Addition-Wesley Publishing company.